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ELECTRO-ANALOG MODEL EQUIPMENT OF NATIONAL ECONOMIC SYSTEM

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This invention relates to electro-analog model equipment for national economic systems, and more particularly to electric devices whose circuits are designed to bring about electric phenomena which are analogous with those in a national economy, the inter-relations and characteristics of the electrical components in such a circuit being arranged to be analogous with those of the economic elements because of which dynamic phenomena take place in the operation of the national economy.

National economic phenomena can be construed as processes of dynamic variations in economic quanta of the elements which are arranged under a national economic system and which are inter-related to each other. Recent research works in the field of theoretical economics have succeded in formulating such inter-relations mathematically in certain functional relationships.

However, it is still difficult to describe thereby what effects would result from variations in an economic quantum on all of the other economic quanta in a national economic system because of multitude of the economic elements and multiformity of the inter-relations therein established. Further difficulties lie in the investigation of problems relating to the effects resulting from variations in more than two economic quanta on all of the other economic quanta in a national economic system, because of the multitude and complexity of factors in the higher orders.

In addition, in the studies of such social phenomena as in national economy, few experimental processes are available for the investigation, contrariwise to natural-scientific studies.

These advantages stand as formidable obstacles against the applications of results obtained from theoretical economic studies for economic policies as well as against studies for further theoretical advancement.

A general picture of a national economic system can be represented in functional relationships mathematically based on the results obtained in recent studies in theoretical economics. Therefore, economic phenomena can be described electrically since electrical phenomena, which are analogous with economic phenomena in such functional relationships, are obtainable by means of electrical circuits.

This invention embodies principles whereby, with an electrical circuit, the components of which are arranged so that electrical quanta in such parts are varied in analogue with variations in quanta in the economic elements in consideration, the interlinking effects of various quantitative variations in the economic elements thereon can be described by means of such reading devices as voltmeters, current meters, oscilloscopes, recorders, etc. which are connected to such terminals in the electrical circuit as to develop variations in voltage or current which are analogous with those in the economic elements.

An object of this invention is to provide a device that functions in anologue with a national economic system, the electrical circuits of the device including electrical components whose characteristics are analogous to those

of the economic elements in consideration and which are arranged and combined in analogue in such interlinking relationship as exists in the national economic system.

Another object of this invention is to provide a device capable of describing national economic phenomena under optional conditions by measuring what effects might be brought by quantitative dynamic variations in one or more such electro-analog components on the other remaining components with an analogue method.

A further object of this invention is to provide a device that may be useful for minimizing difficulties in theoretical descriptions of national economic phenomena and also for making preliminary or pilot experimentations for establishing national economic policies and for theoretical studies.

For a better understanding of this invention, reference is taken to the accompanying drawings, wherein Fig. 1 is a diagrammatic representation of a device for considering a national economic system showing the inter-relationships of economic elements thereof for investigation of dynamic phenomena therein; Fig. 2 is a diagram in which the dynamic inter-relationships in the economic system as shown in Fig. 1 are formulated mathematically in functional relationships; Fig. 3 is a connection diagram of an electro-analog device embodying this invention, by means of which dynamic inter-relationships in economic phenomena can be described with an electro-analog method; Fig. 4 is a diagram in which functional relationships between the elements of the circuit in Fig. 3 are formulated mathematically to prove, as will be understood, that the electrical circuit is constructed analogous with the national economic system with respect to the functions mathematically given in Fig. 2; Fig. 5 is a connection diagram of an insulated coupler which is included in the circuit given in Fig. 3, and Fig. 6 is a connection diagram of another type of insulated coupler which is also employed in this circuit.

An example of a general picture of national economic system, basing on which this invention can be embodied, is shown in Fig. 1 for which explanation is set forth below.

A national economic system can be construed in various ways as the point of view differs, but it can be outlined as shown in Fig. 1 when the problem is to investigate quantitative dynamic variations in a national economic system in the most simplified scheme. The general picture of a national economic system illustrated in Fig. 1 is based upon the theoretical explanations made by F. V. Hyake relating to the production structure and also those made by J. M. Keynes relating to the monetary structure of a national economic system, both of whom have leading positions in the field of theoretical economics.

As shown in Fig. 1, a national economic system can be construed as consisting of Capital Goods Production Group PGP, Consumer Goods Production Group CGP, Trading Group COM, Consumer Group CSM, Banking Organ BK and Investment Channels INV. The first three groups contain the productive labours La_1, La_2 and La_3, respectively, which are accompanied by an individual inertia property U_1, U_2 and U_3, and the stocks of goods St_1, St_2 and St_3 respectively, the goods being the products from the labours La_1, La_2 and La_3 respectively. The consumer group is represented as consisting of the consumption CN having an inertia property U_4.

The inter-connections of such labours, inertias and stocks in the productive groups (including the trading group) and the consuming group are shown in Fig. 1 by the dotted lines which represent monetary flow and by the solid lines which represent flow of goods. Incomes from the capital flow production group, consumer goods production group and trading group are

aggregated into the national income Y, of which one part flows as indicated by the arrow S into the banking organ BK for saving and the other part flows as indicated by the arrow C into the consumer group CSM for consumption.

The saved part of national income is further directed into the channels of investments INV through the banking organ BK, and investments are directed into any of the productive groups PGP, CGP and COM in the forms of "Autonomous Investment" G_1, G_2 and G_3 and "Induced Investment" I_1, I_2 and I_3 respectively. Magnitudes of the induced investments I_1, I_2 and I_3 are influenced by the acceleration effects Acc_1, Acc_2 and Acc_3 respectively, which cause a magnified change in the rate of demand for goods being produced in earlier productive stages when a change in the rate of demand for the goods being produced in a productive stage takes place.

Such functions as set out above may be clarified furthermore by the detail explanations which will be hereinafter described with reference to Fig. 3.

The dynamic interrelationships between the economic elements, which, by way of example, constitute the national economic system, can be formulated in such functional relationships as are mathematically set out in Fig. 2 from the macro-economic point of view.

It is an essential basis of this invention that dynamic experimentations and measurements on national economic phenomena can be made by means of an electrical circuit in which electrical phenomena, which are analogous to those in the functional relationships in Fig. 2, are obtainable. Referring to the national economic system mentioned in Fig. 1 for example, it can be shown that the operation of an electrical circuit such as shown in Fig. 3 results in electrical phenomena which can be expressed as the functional relationships set out in Fig. 4, which stand for those in Fig. 2 completely in analogue. Therefore, it can be understood that, under optional conditions, the description of effects, which should be brought about by quantitative dynamic variations in one or more economic elements, on the others, of which such a national economic system as shown as in Fig. 1 consists, can be made by means of an electrical circuit such as shown in Fig. 3 and by the measuring of analogous effects which should be caused by similar quantitative dynamic variations in the electro-analog components, of which the electrical circuit consists, of dynamic variations in voltage or current.

The analogous relationships for the corresponding economic and electrical items are given in Table 1 with respect to the symbols used in Figs. 1, 2, 3 and 4.

TABLE 1

Item	Economic Item		Electrical Item	
	Symbol	Subject	Symbol	Subject
(0)			E_0	(Supply voltage).
(1)	q_1	Rate of change in flow of capital goods, §1.	i_1	Current standing for q_1.
(2)	r_1	Resistivity or marginal cost against capital goods production.	R_1	Resistance standing for r_1.
(3)	U_1	Inertia in capital goods production.	L_1	Inductance standing for U_1.
(4)	q_1r_1	Unit production cost level of capital goods.	i_1R_1	Voltage standing for q_1r_1.
(5)	$U_1\frac{dq_1}{dt}$	Unit profit level in capital goods production.	$L_1\frac{di_1}{dt}$	Voltage standing for $U_1\frac{dq_1}{dt}$.
(6)	$q_1r_1+U_1\frac{dq_1}{dt}=P_1$	Unit price level of capital goods.	$E_0-\left(i_1R_1+L_1\frac{di_1}{dt}\right)=E_1$	Voltage standing for P_1 in reciprocal order.
(7)	S_1	Inflexibility of stock of capital goods.	C_1	Capacitance standing for S_1.
(8)	qs_1	Flows in and from stock of capital goods, §2.	ic_1	Current standing for qs_1.
(9)	$\int_0^t qs_1dt$	Amount of capital goods in stock at the time t.	$\int_0^t ic_1dt$	Charge standing for $\int_0^t qs_1dt$.
(10)	$\frac{1}{S_1}\int_0^t qs_1dt=\frac{1}{P_1}$	In dynamic equilibrium with (6)	$\frac{1}{C_1}\int_0^t ic_1dt=E_1$	In dynamic equilibrium with (6).
(11)	$q_1\left(q_1r_1+U_1\frac{dq_1}{dt}\right)$	Rate of change in income from capital goods production, §3.	$i_1\left(i_1R_1+L_1\times\frac{di_1}{dt}\right)$	Power standing for $q_1\left(q_1R_1+U_1\frac{dq_1}{dt}\right)$.
(12)	q_2	Same as in (1) but regarding consumer goods.	i_2	Current standing for q_2.
(13)	r_2	Same as in (2) but regarding consumer goods.	R_2	Resistance standing for r_2.
(14)	U_2	Same as in (3) but regarding consumer goods.	L_2	Inductance standing for U_2.
(15)	q_2r_2	Same as in (4) but regarding consumer goods.	i_2R_2	Voltage standing for q_2r_2.
(16)	$U_2\frac{dq_2}{dt}$	Same as in (5) but regarding consumer goods.	$L_2\frac{di_2}{dt}$	Voltage standing for $U_2\frac{dq_2}{dt}$.
(17)	$P_2+\left(q_2r_2+U_2\frac{dq_2}{dt}\right)=P_2$	Same as in (6) but regarding consumer goods.	$E_1-\left(i_1R_2+L_2\frac{di_2}{dt}\right)=E_2$	Voltage standing for P_2 in reciprocal order.
(18)	S_2	Same as in (7) but regarding consumer goods.	C_2	Capacitance standing for S_2.
(19)	qs_2	Same as in (8) but regarding consumer goods.	ic_2	Current standing for qs_2.
(20)	$\int_0^t qs_2dt$	Same as in (9) but regarding consumer goods.	$\int_0^t ic_2dt$	Charge standing for $\int_0^t qs_2dt$.
(21)	$\frac{1}{S_2}\int_0^t qs_2dt=\frac{1}{P_2}$	In dynamic equilibrium with (17)	$\frac{1}{C_2}\int_0^t ic_2dt=E_2$	In dynamic equilibrium with (17).
(22)	$q_2\left(q_2r_2+U_2\frac{dq_2}{dt}\right)$	Same as in (11) but regarding consumer goods.	$i_2\left(i_2R_2+L_2\frac{dq_2}{dt}\right)$	Power standing for $q_2\left(q_2r_2+U_2\frac{dq_2}{dt}\right)$.
(23)	q_3	Same as in (1) but regarding consumer goods being handled by trading group.	i_3	Current standing for q_3.
(24)	r_3	Same as in (2) but regarding consumer goods being handled by trading group.	R_3	Resistance standing for r_3.
(25)	U_3	Same as in (3) but regarding consumer goods being handled by trading group.	L_3	Inductance standing for U_3.
(26)	q_3r_3	Same as in (4) but regarding consumer goods being handled by trading group.	i_3R_3	Voltage standing for q_3r_3.
(27)	$U_3\frac{dq_3}{dt}$	Same as in (5) but regarding consumer goods being handled by trading group.	$L_3\frac{di_3}{dt}$	Voltage standing for $U_3\frac{dq_3}{dt}$.

TABLE 1—Continued

Item	Economic Item		Electrical Item	
	Symbol	Subject	Symbol	Subject
(28)	$P_3+\left(q_3r_3+U_3\frac{dq_3}{dt}\right)=P_3$	Same as in (6) but regarding consumer goods being handled by trading group.	$E_3-\left(i_3R_3+L_3\frac{di_3}{dt}\right)=E_3$	Voltage standing for P_3 in reciprocal order.
(29)	S_3	Same as in (7) but regarding consumer goods being handled by trading group.	C_3	Capacity standing for S_3.
(30)	q_{s3}	Same as in (8) but regarding consumer goods being handled by trading group.	iC_3	Current standing for q_{s3}.
(31)	$\int_0^t q_{s3}dt$	Same as in (9) but regarding consumer goods being handled by trading group.	$\int_0^t iC_3dt$	Charge standing for $\int_0^t q_{s3}dt$.
(32)	$\frac{1}{S_3}\int_0^t q_{s3}dt=\frac{1}{P_3}$	In dynamic equilibrium with (28)	$\frac{1}{C_3}\int_0^t iC_3dt=E_3$	In dynamic equilibrium with (28).
(33)	$q_3\left(q_3r_3+U_3\frac{dq_3}{dt}\right)$	Same as in (11) but regarding consumer goods being handled by trading group.	$i_3\left(i_3R_3+L_3\times\frac{di_3}{di}\right)$	Power standing for $q_3\left(q_3r_3+U_3\frac{dq_3}{dt}\right)$.
(34)	r_4	Rate of change in flow of goods to consumption.	i_4	Current standing for r_4.
(35)	r_4	Resistivity against consumption	R_4	Resistance standing for r_4.
(36)	U_4	Inertia in demand	L_4	Inductance standing for U_4.
(37)	$P_4+U_4\frac{dq_4}{dt}=q_4r_4=P_4$	Consumption price level	$E_4-L_4\frac{di_4}{dt}=i_4R_4=E_4$	Voltage standing for P_4 in reciprocal order.
(38)	$q_4^2r_4$	Rate of change in consumption § 4	$i_4^2R_4$	Power standing for $q_4^2r_4$ in reciprocal order.
(39)	$(11)+(22)+(33)=Y$	Rate of change in national income, § 5	$(11)+(22)+(33)=Y$	Power standing for Y.
(40)	$\alpha\{(11)+(22)+(33)\}$	Rate in change in consumption, § 6	$\alpha\{(11)+(22)+(33)\}$	Power standing for αY.
(41)	$(1-\alpha)\{(11)+(22)+(33)\}$	Rate of change in saving, §7	$(1-\alpha)\{(11)+(22)+(33)\}$	Power standing for $(1-\alpha)Y$.
(42)	B	Capacity of banking organ	C_b	Capacity standing for B.
(43)	$\frac{1}{B}\int_0^t q_bdt=\frac{1}{\gamma}$	Reciprocal of rate of interest	$\frac{1}{C_b}\int_0^t iC_bdt=E_\gamma$	Voltage standing for rate of interest in reciprocal order.
(44)	q_b	Monetary flow in or from B	iC_b	Current standing for q_b.
(45)	β_1	Acceleration in inducing investment directed to capital goods production.	β_1	To be set by $Racc_1$.
(46)	$\beta_1\left(M_2U_2\frac{dq}{dt}\right)=I_1$	Induced investment directed to capital goods production.	$\beta_1\left(M_2L_2\frac{di_2}{dt}\right)=E_xV_1$	Grid voltage to V_1 standing for I_1.
(47)	β_2	Acceleration in inducing investment directed to consumer goods production.	β_2	To be set by $Racc_2$.
(48)	$\beta_2\left(M_1U_2\frac{dq_2}{dt}\right)=I_2$	Induced investment directed to consumer goods production.	$\beta_2\left(M_2L_2\frac{di_2}{dt}\right)=E_xV_2$	Grid voltage to V_2 standing for I_2.
(49)	β_3	Acceleration in inducing investment directed to trading group.	β_3	To be set by $Racc_3$.
(50)	$\beta_3\left(M_4U_4\frac{dq_4}{dt}\right)=I_3$	Induced investment directed to trading group.	$\beta_3\left(M_4L_4\frac{dq_4}{dt}\right)=E_xV_3$	Grid voltage to V_3 standing for I_3.
(51)	G_1	Autonomous investment directed to capital goods production.	EG_1	To be set by $Raut_1$.
(52)	G_2	Autonomous investment directed to consumer goods production.	EG_2	To be set by $Raut_2$.
(53)	G_3	Autonomous investment directed to trading group.	EG_3	To be set by $Raut_3$.

Notes

§ 1. q is $\frac{dQ}{dt}$, where Q is amount produced. Same as to (12), (23) and (34).

§ 2. q_s is $\frac{dQ_s}{dt}$, where Q_s is amount in stock. Same as to (19) and (30).

§ 3. $q\left(qr+L\frac{dq}{dt}\right)$ is $\frac{dy}{dt}$, where y is amount of income, same as to (22) and (33).

§ 4. $q_4^2r_4$ is $\frac{dC}{dt}$, where C is amount of consumption.

§ 5. Means $\frac{dy}{dt}$.

§ 6. Means $\alpha\frac{dy}{dt}$, where α is propensity to consume.

§ 7. Means $(1-\alpha)\frac{dy}{dt}$.

8. At a state of equilibrium, i_1, i_2, i_3 and i_4 become equal each other.

9. At a state of equilibrium, $L\frac{di}{dt}$ is zero.

10. In Figure 3, it is to be understood that goods and moneys flow in opposite directions from each other.

Fig. 3 is a connection diagram, by way of example, of an electrical circuit embodying this invention and the electrical functions of this circuit can be formulated mathematically as shown in Fig. 4. Analogous relationships between Figs. 1 and 2 and Figs. 3 and 4 will be understood more clearly by referring to the itemized collations in Table 1 and the explanations which will appear later with respect to Fig. 3.

The electrical circuit shown in Fig. 3 is so arranged and connected as to have functions analogous with the economic system given in Fig. 1. The main electrical circuit consists of a multistage direct current circuit, each of the stages being composed of a resistance, an inductance and a capacitance and mediated by the capacitance. A direct current voltage is supplied to the input terminal D.C. and the circuit is loaded with the resistance R_4. The first stage in view of the terminals D.C. which consists of the resistance R_1, inductance L_1 and capacitance C_1, stands for the capital goods production group PGP; the second stage consisting of the resistance R_2, inductance L_2 and capacitance C_2 stands for the consumer goods production group CGP; the third stage con-

sisting of the resistance R_3, inductance L_3 and capacitance C_3 stands for the trading group COM; and the last stage consisting of the resistance R_4 and inductance L_4 stands for the consumer group CSM and terminates this circuit. V_1, V_2, V_3 and V_4 are vacuum tubes which belong to the said stages respectively and their direct current anode resistances are operated as the resistances R_1, R_2, R_3 and R_4 respectively. The resistances R_1, R_2 and R_3 stand for the resistivities which take place against the productive labours (including both human and mechanical labours) in the respective stages. (As generally noted, resistance there is against any labours at work, and in an inverse sense, the labour can be defined as the overcoming of such a resistance. The overcoming of such a resistance, or disposal of labours, requires an expense, and a greater resistance requires more expense.) Such a productive labour is accompanied by an inertia, and the inductances L_1, L_2 and L_3 stand for such an inertia property in the respective stages. (A greater inertia results in more time elapsing in changing the volume of the production.) The capacitances C_1, C_2 and C_3 stand for the inventory functions for the goods produced or handled in the respective stages. (A greater capacitance or inflexibility of stocks results in a greater stability in the level of the supplies as well as prices thereof against a sudden change in level of the demand or production.) Further explanation of the functions in each stage is made in the following.

(A) *Functions of the R_1—L_1—C_1 circuit standing for the capital goods production group PGP*

The resistance R_1 stands for the resistance against the labour or marginal cost of producing capital goods and the electric current i_1 represents the rate of change in the flow of capital goods, the voltage drop i_1R_1 standing for the unit production cost for the capital goods. The rate of change in the flow of capital goods, which appears in this illustration, means

$$\frac{dQ_1}{dt}$$

where Q_1 is the amount of capital goods produced or contrariwise

$$Q_1=\int_0^t i_1 dt$$

Therefore the rate of change in the total cost for such production is expressed by $i_1(i_1R_1)$ and the sum of cost in a certain period is represented by

$$\int_0^t (i_1^2R_1)dt$$

which constitutes majority of the income from such production. The capacitance C_1 stands for the inflexibility of the stock of capital goods, and the charging or discharging current i_{C_1} to or from the capacitance C_1 represents the rate of change in the flow of capital goods to or from the stock, which influences the amount of capital goods in stock, viz.

$$i_{C_1}=\frac{dQ_{c_1}}{dt}$$

where Q_{c_1} is the amount in stock or contrariwise

$$Q_{c_1}=\int_0^t i_{c_1}dt$$

In addition, it is understood in our modern economy that price of goods is a function of the amount of the goods in stock and a determinant for the functional relationship is the inflexibility of the stock. Therefore, the price level of capital goods P_1 can be represented by a functional relationship such as

$$P_1=\frac{S_1}{\int_0^{t_s} s_1 dt}$$

and the analogous relationship between P_1 and E_1 is established in reciprocal order viz.

$$P_1=\frac{1}{E_1}$$

or vise versa, as the voltage E_1 being developed across terminals of the capacitance C_1 is in such a functional relationship as

$$E_1=\frac{1}{C_1}\int_0^t i_{c_1}dt$$

In an equilibrium state, where

$$L_1\frac{di_1}{dt}=0$$

and $i_{C_1}=0$ at the same time $(E_0-i_1R_1)$, where E_0 is the power supply voltage to the terminals D.C., becomes equal to the voltage across terminals of the capacitance C_1, or $E_0-i_1R_1=E_1$, and this stands for the price level of capital goods in reciprocal order or for $1/P_1$.

Assuming now that the load on the R_1—L_1—C_1 stage presented by the following R_2—L_2—C_2 stage, which stands for the consumer goods production group CGP, becomes heavy suddenly for some reason, or that the demand for capital goods by the consumer goods production group CGP are increased suddenly for some reason, then the initial increase in the current to the R_2—L_2—C_2 circuit takes place under the favour of the capacitance C_1 because the two circuits are mediated by the capacitance C_1 which stands for the stock function for the capital goods, and no sufficient increase in the current through the resistance R_1 can take place until a certain time elapses according to the magnitude of momentum due to the inductance L_1, viz. an instantaneous increase in the production of capital goods is restrained by the momentum due to the production inertia U_1, and the supply of capital goods to meet the increased demand is made under the favour of the stock Q_{s_1} until the production be raised sufficiently.

The capacitance C_1 is so large that while undertaking the supply of the increment in the flow of capital goods to the consumer goods production group CGP, no sudden change in the voltage across terminals of the capacitance C_1 takes place, viz. no suddent change in the price level of the capital goods takes place.

However, as the charge in the capacitance C_1 decreases, viz. the stock of capital goods decreases, a voltage

$$L_1\frac{di_1}{dt}$$

is induced in the inductance L_1 and until the current i_1 increases sufficiently such an induced voltage complements the lack of equilibrium between $(E_0-i_1R_1)$ and E_1, viz. the pricewise lack of equilibrium between the supply and demand is complemented with such an inducement.

Such an equilibrium relationship can be expressed by such an equation as

$$E_1=E_0-\left(i_1R_1+L_1\frac{di_1}{dt}\right)$$

and it can be understood that E_1 is reduced while the current i_1 does not increase sufficiently because of the momentum due to the inductance L_1, as compared with the level at an equilibrium where the load against the R_1—L_1—C_1 circuit has not yet become heavy, viz. until the production of capital goods meets such an increase in demand, the price level of capital goods goes up, and it can be seen that

$$L_1\frac{di_1}{dt}$$

stands for an income which is more than the production cost, or for a profit. However, if no change takes place